Measuring the zonal error of a mirror with two-slit interferometric method

Original authors Tapio Korhonen and Johannes Kultima, Tuorla Observatory
Published in Tähtitaivas, issue 33, Turun Ursa, 1973
Translated by Jani Achren, 2012, <jani.achren@incidentangle.fi>

For an amateur, the biggest problems in polishing a parabolic mirror must be the measurement of the surface contour. The most commonly used Foucault knife edge test is mostly suitable to the overall evaluation of the shape. Quantitative analysis using knife edge requires means not often available to amateurs, and even then the precision of the measurements is not convincing.

Measurement technique based on the interference of light combines high precision and simplicity of means, and are therefore most suitable for amateurs. The purpose of this text is to give directions for measuring the shape of the mirror using interference of two slits. The technique is equally valid for use with refractive elements also, but since amateurs rarely attempt manufacturing them, only mirrors are considered in the following text.

Väisälä interferometric technique is based on the measurement of a wavefront emanating from a light source. Light emanating from a point source at infinity can be though as progressing as a wavefront with a flat shape - at all points of the wavefront, the radiation is of the same phase. After progressing through a perfect objective, the wavefront is spherical, and the image will be formed at it's center. Due to errors of the objective the wavefront is not spherical, and measuring this wavefront is the same as measuring the errors of the objective.

Let's examine the the principle of the measurements with illustration 1. *P* is the spherical wave that has center and the location of the image at *O*. *P'* is the true wavefront that we need to measure. For measuring it, an obstruction is placed on the light path with slits positioned at H_1 and H_2 on the same diameter of the objective. Due to the wave nature of light, interference lines are formed into the focal plane. The middle one, which is the strongest, appears at *U*, to where the distance from wavefront points H_1 and H_2 is exactly the same.

Lets mark the wavefront errors as h_1 and h_2, respectively, and the space *OU* by *u*. If the focal length of the objective is *F* and the distance between the center of slits is *D*, then

$$h_2 = h_1 + u\frac{D}{F}$$

By moving the slits along the diameter with steps of *D* and measuring the location of the interference maximum, the wavefront errors with respect to a spherical wave can be calculated. (The spherical reference wavefront is dependent on the location of the measuring coordinate's origo.)

This should be enough about the theory of the method. In the following, guidelines to measurements and and calculations are given with examples. One may derive the the formulas if one so desires.

For the measurements, a point source of light, eyepiece and a mask with slits in front of the mirror is needed. The most accurate measures can be taken with an eyepiece microscope. Since this type of device is readily available only to few amateurs, a method of reduced precision but without a microscopic eyepiece is presented.

As a light source, one may use a matted light bulb, and light is directed to the mirror via a pinhole. A good method of making a pinhole is to stick a needle through many layers of aluminum sheets, from where the most suitable is chosen with the help of a magnifying lens. The light source is positioned close to the center of curvature of the mirror.

One needs an eyepiece with relatively strong magnification (such as 10mm) to see the interference lines. An eyepiece must be purchased for the telescope anyway. In emergencies, a magnifying lens is also acceptable. The eyepiece must be supported so that it won't move during the measurements. The best is to position the light source and the eyepiece to the same holder as close as possible to each other. Distance can be reduced if the light of the source is reflected with a mirror or a prism into the pinhole. This way the light source may be positioned a bit further. If the measurements are done along the horizontal diameter, the best place for the light source is above the eyepiece. This way the measurements won't show large error due to the fact that both cannot be positioned to the center of curvature.

The mirror will be covered with a mask that has slits equally spaced along the diameter. Slits can be stationary, such as in illustration 2a. A strip showing only two slits at a time is moved in front of the row. Another choice is to make a hole the size of the objective to the mask, and put slits into a strip, like in illustration 2b. In this case the strip must be equipped with scale with the separation of slits as a step size. Correspondingly, a reading mark on the mask must be prepared. During measurements, the strip is moved one step size at a time.

The slits must be placed so that one is coincident with the center of mirror. The number of slits is determined by how many measurement points are wished. Usually one can use slit separation that is 1/100 of focal length. The outermost slits must also be coincident with the edge of the mirror.

In the absence of eyepiece microscope, the measurements can be done with "an auxiliary interferometric pattern", which is mentioned in Y. Väisälä's dissertation paper. This method was used in the testing of Schmidt telescope of Kvistaberg observatory[1] by Y. Väisälä and L. Oterma. In addition to measuring slits, the mask is equipped with a two slit arrangement with the same slit distance as with measurement slits. The two slits will be positioned parallel to the measuring slits and as far away from them as possible, as in illustration 3. The point source will then produce two interference patterns.

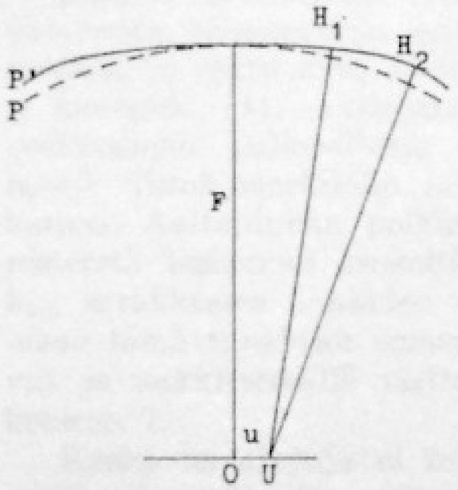
The measurement is done by approximating the distance between the center line of the measurement interferometric pattern to the scale provided by the auxiliary interferometric pattern, with the accuracy of 1/10 of spacing width. Illustration 4 describes examples of mutual positions and interpretations taken from them. The auxiliary pattern is above, marked by five lines. The center lines are drawn longer for clarity. In reality they are just brighter than others. Let the first reading of the center auxiliary line be 0 and the positive axle from left to right as per illustration 4a. If the center line of the measuring pattern exits the auxiliary's pattern, the measurement can be read via the line next to the center line as per illustration 4c.

The appropriate shape of the slits is a square and the distance between the centers of slits (=slit spacing) twice the length of the square. This way pattern consists of three clear lines, whereas more would create confusion. For the auxiliary pattern, the slits can be made thinner, to bring out more lines. The appropriate line amount is five or seven. To increase brightness of the patterns, the slits may have to be lengthened. This may be necessary with measurement slits as well. One should experiment to find the most appropriate slit length to fit one's own instruments.

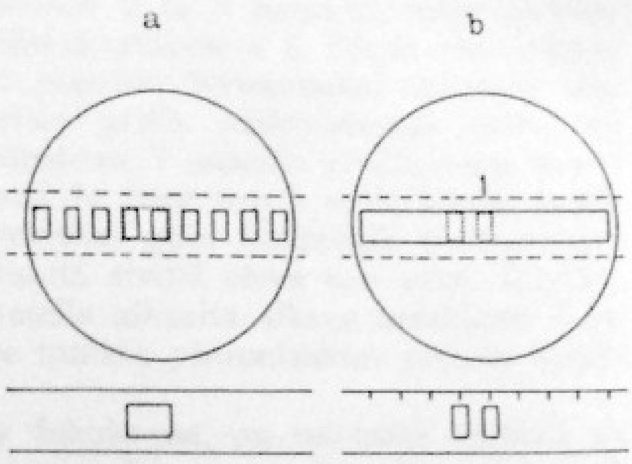
When the eyepiece is focused to the point source, the interference patterns will overlap. Therefore the measurements must be done with a deviation from focus. This will create an error between a spherical wavefront and the incoming wavefront, but it can be taken into account in calculations. In any case, the deviation from focus must be kept minimum while maintaining the visible separation of patterns.

The measurements takes place in a

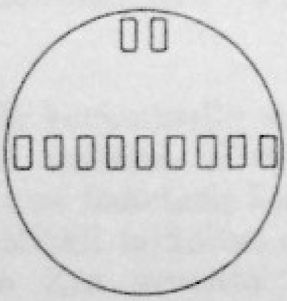
1: http://www.astro.uu.se/history/Schmidt.html



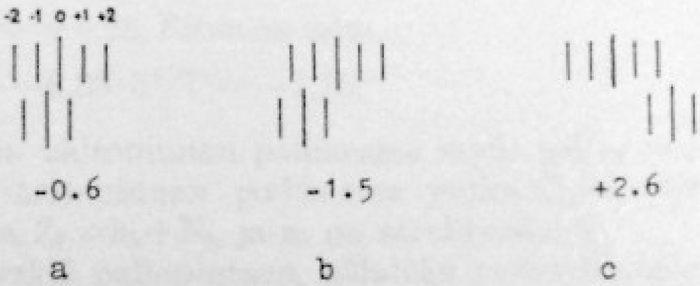
Kuva 1
Illustration 1



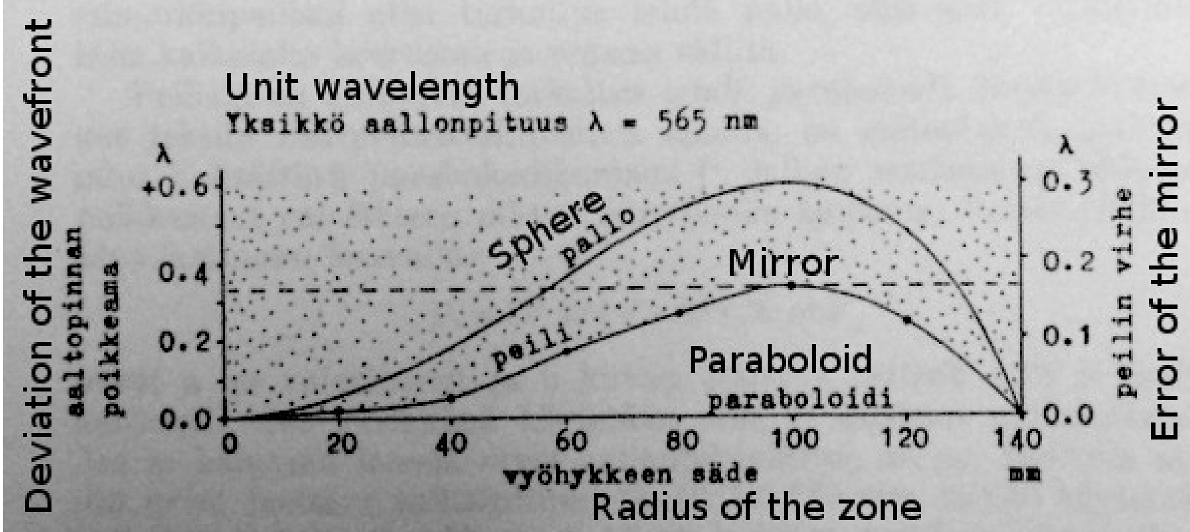
Kuva 2
Illustration 2



Kuva 3
Illustration 3



Kuva 4
Illustration 4



Kuva 5
Illustration 5

darkened room with no drafts, definitely nowhere near heat sources or windows, as air currents make the interference slits vibrate. While observing, eyes must be as relaxed as possible, since changes in its focus makes the locations between interferometric lines change slightly. Tiny movements of the mirror are inconsequential, since the auxiliary slits will move with the mirror. However, the distance between eyepiece and mirror must not change.

In the next example, flow of the observations and the calculations will be further clarified. Mirror defects are assumed to be zonal defects, making the surface symmetric along its optical axis. Observations and calculations will be marked on a spreadsheet, a very useful help while doing your own observations. At the same time the observing order should be maintained so that there won't be sign confusions.

Observations were done inside the focus with a mirror of diameter 300 mm and focal distance of 2063 mm. Slit spacing was 20 mm with 15 slits. The outermost slits were therefore at a distance of 140 mm from the center of the mirror. The slits were numbered starting from the center slit as 0, 1, 2, … , 7. These are represented with a letter *k* at column 1.

At start, the slit cover is positioned so that the leftmost two slits only are visible. The position of the interference pattern is compared to the reference interference pattern, and the distance between center lines is observed. Then the slit cover is moved a distance of slit spacing to the right and second observation is done, etc. The observations are recorded to column 2 of the spreadsheet, between the lines corresponding to the number of the slits used in observation.

After the mirror's left side observations are complete, observations proceed to the right side, marking the readings to column 3. This way the observations done at the same distance from the center are recorded to the same line in the spreadsheet. When the rightmost observation is complete, the measurement is sufficient in principle. For verification, the observations are repeated in reverse order, moving the slit cover from right to left. Observations will be marked on columns 4 (mirror's right side) and 5 (left side).

The calculations are done row by row. The sum of values from column 4 and 5 is deducted from the sum of values from column 3 and 4, divided by four and marked on column 6. These are values *u* on the first formula. To calculate the shape of the wavefront, the deviation is assumed to be zero at the center, h_0=0. This is marked on column 7, on the same line with slit 0. The wavefront deviation h_k (at a slit with a distance of *k* from the center) is calculated by adding the value of *u* (between the lines *k* and *k-1* from column 6) to the deviation at h_k*-1*. In practice this means summing up the values in column 6 starting from down, and marking the intermediate results to column 7.

Because the observations were not done at focus, a quadratic correction *N* must be done. If the deviation of wavefront at the furthermost slit *v* is h_v, quadratic correction at slit *k* is

$$N_k = -h_v \frac{k^2}{v^2}$$

In our example *v*=7 and h_7=+6.28. Therefore

$$N_3 = -6.28 * \frac{3^2}{7^2} = -1.15$$

With this correction the wavefront deviation at the edges can be set to zero. The corrected wavefront deviation with regards to spherical wave is calculated with a formula

$$Z_k = h_k + N_k$$

and is marked on column 9.

If the aim is a spherical surface, one can see the status of the mirror straight from the values of *Z*. The unit is wavelength of the light being used, which is 565 nm when using white light, as per experience. The mirror errors are half of the wavefront errors. Wavefront deviation is defined as positive when the wavefront is behind the spherical wavefront to the direction of the propagation. In the case of negative deviation one must remove glass from the mirror. If the mirror in the example is supposed to become spherical, glass must be removed everywhere between the center and the edge.

But the mirror is supposed to become a paraboloid. Because the observations were done near the center of curvature, a parabolic correction *P* must be added to the wavefront deviation *Z* in order to get wavefront deviation as if the light source had been in infinity. The parabolic correction is calculated with formula

$$P_k = D^4 \frac{k^2 (v^2 - k^2)}{8 a b F}$$

where a is the distance of light source and b image distance from the mirror and *F* focal length. If the unit is mm, unit of *P* is also mm. If you need to express it also as wavelengths, you must divide the values from the above formula by the wavelength (0.000565mm if you're using white light). Paraboloid correction as wavelengths are calculated in column 10. By adding them to *Z*, one gets deviations of the wavefront for a light source at infinity (column 11).

As the values of *K* are positive, there is too little glass between the center and the edge. One cannot simply add glass there, so the center and the edges must be polished down. The mirror is still under-corrected. The error of the mirror is half the deviation of the corrected wavefront *K*. The paraboloid correction *P* can be thought of as wavefront from a source at infinity when using a spherical mirror. Image 5 describes wavefront deviations from this mirror and a spherical mirror of equivalent size. If you wish to describe the mirror surface using these curves, you must envision the glass material as being above the curve (the shaded area in image 5). The shape of the mirror is approximately half way between a spherical and paraboloid. In order to make it full paraboloid, one must remove glass until the dotted line.

If one has access to ocular microscope, one can make much more accurate observations. Auxiliary interferometric slits are of course, unnecessary and the focusing must be very accurate. Observations are now values of the micrometer and the calculations will go on as previously. Deviations of the wavefront *Z* have to be multiplied by

$$D \frac{r}{b} \quad ,$$

where *r* is the rise in micrometer, etc. The units are in mm, so the final calculations must also be done with the same unit.

When using the auxiliary interferometric slit method, it is advisable to do the measurements both inside as well as outside the focus in order to counter some of the measurement errors. Also it is recommended to use at least two diagonals for the measurements.

If the interference lines are not visible, the light source is probably too weak. Since making larger slits is not advisable, it's best to use brighter lamp or use optics to concentrate the beam into the pinhole. Also lengthening the slits brightens the lines. However, the height must not be more than two or three times the slit spacing. Increasing the slits has also the benefit of reducing the height of the interference lines, thus enabling the lines form the slits to get closer to the auxiliary lines.

If the process wasn't presented clearly enough, or other difficulties arise, one may contact the writers at the Turku University, Tuorla observatory, for further instructions.

Johannes Kultima and Tapio Korhonen.